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Tutorial 3 – Water pollution (based on the final exam 09/10)

Consider I cities located around a lake. Each city pumps water inside the lake to satisfy the water demand by urban users. The pumping costs are assumed to be null. Let x_i be the water consumed by a city i. By consuming the water the urban users pollute it. Let q_i be the pollution rate generated by a city i in solid waste equivalent. Used water is collected by each city and sent to a sanitation plant for water cleanup. Let a_i be the amount of pollution eliminated by the sanitation plant of a city i and let $z_i = q_i - a_i$ be the net pollution rate. Net pollution is then discharged into the lake. Eliminating pollution incurs an operating cost of the sanitation plant $C(a_i)$, where $C(0) = 0$, $c(a_i) = dC(a_i)/da_i > 0$, $c(0) = 0$ and $c'(a_i) > 0$ with $\lim_{a_i \to q_i} c(a_i) = \infty$. This last assumption ensures that no city will eliminate completely its pollution and so $z_i > 0$ for all the cities. Note also that all cities use the same sanitation technology and hence share the same cost function.

The urban users can make efforts to control their gross pollution rate q_i but these efforts are costly. The gross surplus function of urban users of city i may be defined as $V(x_i, q_i)$ where $v^x(x_i, q_i) \equiv \partial V(x_i, q_i)/\partial x_i > 0$ if $x_i < \bar{x}_i(q_i)$ and equal to 0 if $x_i = \bar{x}_i(q_i)$, while $v^q(x_i,q_i) \equiv \partial V(x_i,q_i)/\partial q_i > 0$ and equal to 0 if $q_i = \bar{q}_i(x_i)$. Note that all users in all cities share the same surplus function. Moreover the function V verifies:

- $\lim_{x_i \to 0} v^x(x_i, q_i) = +\infty$ and $\lim_{q_i \to 0} v^q(x_i, q_i) = +\infty$. Under these assumptions both $x_i > 0$ and $q_i > 0$.
- $\partial v^x/\partial x_i < 0$, $\partial v^q/\partial q_i < 0$ and $\partial v^x/\partial q_i = \partial v^q/\partial x_i > 0$. Under these assumptions the marginal surpluses are strictly decreasing functions of their arguments.
- Denoting by $V_{xx} < 0$, $V_{qq} < 0$ and $V_{xq} > 0$ the above second order partial derivatives, $V_{xx}V_{qq} - V_{xq}^2 > 0$. Under this assumption, the function $V(x_i, q_i)$ is concave in (x_i, q_i).

Let $Z \equiv \sum_{i=1}^{I} z_i$ be the aggregate pollution discharged into the lake by the cities. This pollution creates an environmental damage for the urban populations living around the lake. Let $D(Z)$ be this damage in welfare terms. Assume that $D(0) = 0$, $D'(Z) > 0$ and $D''(Z) > 0$: the damage is an increasing and convex function of the lake pollution rate.

Q1: *Check that the urban users maximizing their surplus should choose the water amount $\bar{x}(q_i)$ which nullifies their marginal surplus v^x. Define by $W(q_i) = V(\bar{x}(q_i),q_i)$ their optimized surplus with respect to x_i and define accordingly $w(q_i) = dW(q_i)/dq_i$. Check first that $w(q_i) = V^q$. (Hint: Think of the envelope theorem since $v^x(\bar{x}, q_i) = 0$). Check then that under the assumptions given above $w'(q_i) < 0$. (Hint: you should get first the expression of $d\bar{x}/dq_i$ using the fact that $V^x(\bar{x}, q_i) = 0$ and then make use of the concavity of the function V embodied inside item 3 of the assumptions).*

Q2: *The cities agree to delegate the management of the lake pollution to a central water agency. The objective of this authority is to maximize the net surplus of the population living around the lake taking into account the environmental damages. Write down the corresponding maximization problem in terms of the function $W(q_i)$ for the cities. Give the first order conditions for maximization with respect to q_i and a_i. Make an economic comment about these conditions. Check that all cities having identical cost and the same surplus functions, they should pollute the same and make the same pollution abatement effort. You will denote by q and a these common levels and by $Q \equiv Iq$ and $A \equiv Ia$ the corresponding aggregate levels. Show on a graph in the (a, q) plane how the optimal levels of a and q can be determined (Hint: differentiate completely the first order conditions to get two relations linking q to a, $q^q(a)$ and $q^a(a)$ and check that they are increasing functions of a.*

Q3: *The central agency is not in position of imposing to the urban users to reduce their own emissions and it can act only upon a_i the abatement efforts of the cities. What can be said about the environmental effect of such a regulation with respect to a Pareto optimal situation?*

Q4: *Without regulation, $a_i = 0$ and $q_i = \bar{q}_i(\bar{x}_i)$, the cities would make no abatement effort and the users would pollute at the maximum rate. The water agency decides to implement the optimal pollution level by setting a tax depending upon z_i the net pollution level of a city i. This tax is levied upon each city. Remember that cities being identical, the tax rate will be the same for all cities. The city administration is then free to impose a charge upon pollution emissions to the users. Compute the level of the optimal tax and the tariff rate upon q_i the city should impose over the users. Comment on the budget balance of the city.*

Q5: *Being unable to observe the individual pollution emissions, the cities have to set a fee upon water consumption x_i in the city. Compute the optimal level of such a fee. Compare to the previous question results.*